Homework Exam 1 2025-2026

My name and StudentID go here!

Deadline: 21 November 2025, 15:15

This homework exam has 1 question for a total of 9 points. You can earn an additional point by a careful preparation of your hand-in: using a good layout, good spelling, good figures, no sloppy notation, no statements like "The algorithm runs in $n \log n$." (forgetting the $O(..)$ and forgetting to say that it concerns time), etc. Use lemmas, theorems, and figures where appropriate.

Question 1 (9 points)

Let $r \in \mathbb{R}^2$ be a "red" point, and let B be a set of n "blue" points in $\mathbb{R}^2$. You can assume that the points are in general position; meaning that no two points have the same x-coordinate or the same y-coordinate, and that no three points lie on a line. A triangle is "bichromatic" when its vertices are either red or blue, and it has at least one vertex of either color. Develop an $O(n \log n)$ time algorithm to find a maximum area "bichromatic" triangle Δ^* on $\{r\}, B$.

Hint: You can use the following fact. A function $f[1..n] \to \mathbb{R}$ is *unimodal* if (and only if) it has a single (local) maximum. A maximum of f can be computed in $O(T \log n)$ time, where T is the time it takes to evaluate a single value $f(i)$ with $i \in [1..n]$. In particular, using the following function $\textsc{TernarySearch}([1,n], f)$:

function $\textsc{TernarySearch}([a..b], f)$
  $n \leftarrow b - a$
  if $n < 3$ **then** evaluate $f(i)$ for each $i \in [a..b]$ and return $\max_i f(i)$
  else
    $m_1 \leftarrow a + \lfloor n/3 \rfloor$; $m_2 \leftarrow a + \lfloor 2n/3 \rfloor$
    if $f(m_1) < f(m_2)$ **then** $\textsc{TernarySearch}([m_1.., b], f)$
    else $\textsc{TernarySearch}([a..m_2], f)$
    end if
  end if
end function