Homework Exam 2 2025-2026

My name and StudentID go here!

Deadline: 10 December 2025

This homework exam has 4 questions for a total of 90 points. You can earn 10 additional points by a careful preparation of your hand-in: using a good layout, good spelling, good figures, no sloppy notation, no statements like “The algorithm runs in $n \log n$.” (forgetting the $O(..)$ and forgetting to say that it concerns time), etc. Use lemmas, theorems, and figures where appropriate. Your final grade will be the number of points divided by 10. Unless stated otherwise, both randomized and deterministic solutions are allowed. In case you are asked to analyze the running time and your algorithm is randomized, analyze its expected running time.

Question 1 (20 points)

Let $\mathcal{S}$ be a planar subdivision with n vertices, represented as a DCEL. Give *pseudo-code* for an output sensitive algorithm that, given a pointer to a face F, reports all k half-edges that have a vertex in common with F.

Your algorithm should use the 'Twin', 'NextEdge', 'PrevEdge', etc. fields to navigate (i.e. you cannot assume you can directly access a list of vertices). Describe in a few sentences the main idea of your algorithm and analyze its running time.

Question 2

1. *(10 points)* Show that a triangulation of a polygon with $n \geq 9$ vertices with $h > 1$ holes may consist of more than n triangles. That is, describe a construction that, given a number $n \geq 9$, constructs a polygon P_n with n vertices that has a triangulation with more than n triangles.
2. *(15 points)* Prove that any triangulation of a polygon P with n vertices and h holes actually has the same number of triangles.

Question 3

Let $p \in \mathbb{R}^2$ be a point, and let S be a set of n disjoint line segments in the plane. You may assume that the set containing p and all endpoints of the segments in S has no three colinear points (and thus p does not lie on any of the segments).

1. *(20 points)* Develop an algorithm to compute the length of a longest line segment $\overline{pq}$ that does not properly intersect the interior of any segment in S. (Recall that two segments properly intersect if and only if their interiors intersect). If segment $\overline{pq}$ does not exist your algorithm should return ∞. Prove that your algorithm is correct and analyze its running time.

   Note: the number of points rewarded for this question will depend on the running time of your algorithm.
2. *(5 points)* Is your algorithm still correct if the segments in S may intersect? If so, argue why, if not, give an example why not, and describe how to fix it. You do *not* have to argue about the running time of your algorithm in this scenario.

Question 4 (20 points)

Let P be a set of n points in $\mathbb{R}^2$, let z be a point that lies strictly in the interior of the convex hull $CH(P)$, and let ρ be a ray (oriented half-line) that starts in z. Develop an expected $O(n)$ time algorithm that, given P, z, and ρ, can find the edge of $CH(P)$ hit by ρ. Prove that your algorithm is correct and achieves the desired running time. You may assume that no three points in P are colinear, and that ρ contains no points of P.

Note that you are *not* given $CH(P)$ itself.